A Reflection on some aspects of Mathematics and Mathematics Education

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Proceedings of ICME-10 – Topic Study Group 2
New Developments and Trends in Secondary Mathematis Education
Copenhagen, Denmark July 2004

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The main concern of these reflections is the way mathematics and mathematics education is reported in the media and the mathematics classroom practice.

1 Introduction

This paper is a personal reflection and therefore some information about my professional background would be appropriate. I got my graduate degree in Mathematics in 1975. I was a teacher of mathematics at secondary school for 10 years and for the last 15 years I have been working at the University of La Laguna as lecture of mathematics and its didactic. From 1985 to 1995 I was advisor to the secondary school mathematics reform for the Local Educational Authority in the Canary Islands. Now I am Profesor Titular (something like associate professor) of Mathematics Education in a Mathematical Department (Análisis Matemático) a quite unusual situation even for Spanish standards.

Over the last decades many proposals about new mathematical content have been set forth. One of the most important is the NCTM Standards (1989 and 2000 versions). Standards has put together many new and innovative proposals for teaching mathematics at the school level. In my own country, Spain, a reform was launched in the late eighties that shared many goals with the NCTM Standards. We have found there the Problem Solving approach, the focus on Modelling and Communications skills among the most innovative and with an intended potential to modify classroom practices.

2 The 2004 Abel Prize (Communicating Mathematics)

Last March 26th I woke up in the morning with the press heading:

El País (M. Ruiz de Elvira): "*The Norwegian Academy of Science and Letters has awarded authors of one of the great landmarks of twentieth century mathematics with the Abel Price for 2004*. In another place highlighted within the text: *The index theorem is a proof for the impossibility of a situations described in an etching of M.C. Escher (Ascending and Descending)*".

The Abel Prize is intended to give the mathematicians their own equivalent of a Nobel Prize. That morning while browsing internet pages for Abel Prize I found these excerpts:

ABC News in Science (H. Catchpole): "Index theory wins top maths prize. The maths used to understand how fast your coffee cools has been recognised in maths' most prestigious prize". Nature Science updated (M. Peplow): "Maths 'Nobel' awarded. Pair get prize for formula that counts solutions to problems. Singer and Atiyah won half a million pounds for their work". But what is Singer and Atiyah theorem about?

A response to that question is found at the Abel Prize web page. There Professor J. Rognes from University of Oslo wrote a wonderful paper intended to be read for a wider audience: *On Atiyah-Singer Index Theorem.*

Index Theorem:
Let P(f) = 0 be a system of differential equations.
Then analytical index(P) = topological index(P)

Let's have a look at some paragraphs of Professor Rognes explanation:
Modern applications of mathematics usually start out with a mathematical model for a part of reality, and such a model is almost always described by a system of differential equations. To make use of the model one seeks the solutions to this system of differential equations, but these can be almost impossible to find. The new insight of Atiyah and Singer was that it is much easier to answer how many solutions there are. And the answer is expressed in terms of the shape of the region where the model takes place.

When you face a difficult problem try to solve another more simple and related. So instead of finding the solutions of the differential equations we must seek for how many there are. But an unexpected result is the format of the answer: we get not a number but a solution expressed in terms of a geometrical shape. That is quite unusual for our students.

A simple analogue can be looked at triangles and quadrangles in the plane. It can be complicated to find the angles in the corners of some of these figures, but sometimes, before Euclid, someone realised that the sum of the angles in all the corners is always 180 degrees for a triangle, and 360 degrees for a quadrangle. The answer to this question is thus easily given, and depends only on the shape of the figure, namely whether it has three or four vertices.

To get insight into the Index Theorem an analogy with an elementary geometry property is used by Professor Rognes. I think this is a very good example of Mathematics Communication, Connecting mathematics, using Analogy to understand a difficult problem (a strategy within Polya's Problem Solving approach). In a few words, this is a good example of many of the curriculum proposal from the last twenty years reforms brought together to explain an important mathematical result. And last but not least it is a good example of a high quality collaborative work among mathematicians.

All these aspects are not highlighted by the press headings. Otherwise they display the pure anecdotal aspects of the news. Other times the news is worst as it happened when the TIMSS results were released in 1997. Most of the press's heading were: Who's on the top? And nearly all newspapers published the nations' ranking on math highlighting the most competitive aspect. I strongly think that this kind of anecdotal news does not help much to modify the social image of mathematics as a school subject. That social perception of mathematics has much to do with mathematics classroom practice.

3 Mathematics classroom practice (Mathematics Education)

Let's have a look at the actual impact of the reform on classroom practice and we shall try to identify some factors which hindered it.

In Spain, as in many western countries, a new curriculum and a great effort on teacher's development has been implemented from the late eighties. However that effort has not succeeded as expected in changing the current mathematics classroom practices.

How can we describe the current mathematics classroom practice?

There is a general agreement that traditional mathematics school practice is still dominated by rote learning and drill and skill methods. In spite of the efforts made at the time the new reform was implemented, efforts in new curriculum and teacher's professional development, it seems that the existing and traditional practice is too difficult to overcome. I think that one of the main reasons is the current exam system.

The role played by final examinations

The current exam system is an important condition for succeeding in any reform. In Spain at the end of secondary education the students have to pass an exam needed for entering the university. The mathematical component of this exam is still focused in the most algebraic and routing aspect of the mathematical knowledge and it has also time limit and has only a paper and pencil format.

When the new curriculum was introduced, many of the secondary school teachers perceived that these ideas were good but they could not change mathematics classroom practice alone. The classroom practice will not be improved merely by setting the new curriculum content and procedures. Many teachers raised the important question: is the current exam system going to change?

Minor aspects of the current exam system were changed by the authorities so the main objective of teachers is to prepare students to obtain good marks in the final secondary education exam.

Technology

It is a common issue that technology has been said to change the nature of mathematics education but the regular integration of technology in classroom teaching is still quite rare.

As factors that affect a productive future integration of technology in mathematics education and thus could modify and improve classroom practices, infrastructure arrangements, adequate research, curriculum development and teacher training are stated (P. Drijvers, these proceedings). I would also add the teacher's beliefs as an important factor which will affect the future of the relationship between information technologies and mathematics education.

The question is: How can information technologies help to change mathematics teacher's beliefs?

PISA and other international comparative studies

It has been said somewhere (see Turner, these proceedings) that PISA is an international comparative study that has the potential to influence secondary mathematics education policy and practice. It is the very second of those purposes, i.e., practice that would be my concern. For practice I mean classroom practice. That is not only what kind of mathematics students encounter within the classroom but also what students are expected to do and what the teacher's role is, i.e., how they help students to develop their mathematical concept and procedures.

TIMSS is more focused on assessing curricular content while the main purpose of PISA is to assess students' abilities to make use of the knowledge and skills they have developed and accumulated over their time at school in order to solve problems that are largely of a practical kind. PISA and TIMSS seems to me as highly complementary. In that way we would say that while TIMSS assesses what pieces of curricular knowledge are lasting after a long period of time at school, PISA assesses how that knowledge is used to handle situations and challenges of day-to-day life.

PISA levels describing mathematical literacy could influence secondary mathematics policies but while keeping in a paper-and-pencil base for assess I can hardly see how it could influence and modify classroom practices.

4 Conclusion

If we want to improve classroom practice by modifying the current practice I strongly think that we have to change attitudes and beliefs. We have to change teacher's attitudes and beliefs and also the way mathematics practice is perceived in our society. So we need to devise and strategy to modify attitudes and beliefs that are widely shared in society.

References

Drijvers, P. (2004). *The integration of technology in secondary mathematics education: Future trend or utopia?* (in this volume)

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standars for School Mathematics.* Reston. Virginia.

National Council of Teachers of Mathematics. (2000). *Principles and Standars for School Mathematics.* Reston. Virginia.

Turner, R. (2004). *PISA and Secondary Mathematics Education.* (in this volume)