Students' Actions in Open and Multiple-Choice Questions Regarding Understanding of Averages

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Proceeding of the 30th Conference of the International Group for the Psychology of Mathematics Education, Vol 3, 161-168.

Charles Univertity. Prague. Czech Republic.

STUDENTS' ACTIONS IN OPEN AND MULTIPLE-CHOICE QUESTIONS REGARDING UNDERSTANDING OF AVERAGES

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The debate about how to assess students' concepts of averages has given rise to different opinions about the suitability of each form of evaluation. In this work we analyse how students act when solving open-answer questions once they have selected the correct option in interrelated multiple-choice questions. Analysis has led us to note that many students who choose the correct answers in multiple-choice questions were completely unable to demonstrate any reasonable method of solving related open questions. This suggests that multiple-choice questions do not provide precise information about students' knowledge and reinforces the importance of open questions when assessing averages.

Introduction

Interest in research about how students reason when faced with questions involving arithmetic averages is gathering among researchers given the importance of this concept in everyday life. Some studies have shown the teaching-learning of this concept is apparently easy, but understanding of the concept gives rise to tremendous difficulties. For example, Pollatsek, Lima and Well (1981) demonstrate that most students seem to know the average calculation rule or algorithm. However, if these students have merely an instrumental knowledge of averages, they will make foreseeable kinds of mistakes in all questions, except the most obvious. According to Watson and Moritz (2000), for a large number of children, the average is simply a value in the centre of distribution. Study undertaken by Cai (1995) has shown that most students know the mechanism of "adding all together and dividing" which constitutes the simple average calculation algorithm. However, only some of them were able to find an unknown value in a series of data where the average is known. Mokros and Russell (1995) demonstrate various difficulties faced by students in their understanding of averages. In this study we have identified and analysed five different constructions of representation used by students: the average as mode, the average as algorithm, the average as something reasonable, the average as mean point and the average as mathematical point of equilibrium.

The debate on how to assess students' conceptions, has given rise to different points of view about the suitability of each form of assessment. Garfield (2003) describes a questionnaire for assessing statistical reasoning, consisting of twenty multiple-choice questions involving concepts of Probability and Statistics. Garfield (2003) believes that most assessment instruments are centred more on the abilities of calculation or problem solving than on reasoning and understanding. Cobo and Batanero (2004) and Cai (1995) underline the importance of open questions for assessment and suggest that this type of questions be used to examine students' ideas about the concept of arithmetical average and the processes of solving problems. Gal (1995) states that it is difficult to judge fully what a person knows about averages as an instrument for solving problems based on data unless a context is given that would motivate the use of that instrument.

In this work we set out to analyse how students act when faced with open answer questions that are closely related to multiple-choice questions. We wanted to find out if those students who choose the correct options in multiple-choice questions have done so using clear criteria, basing our observations on the students' actions during the process of solving open questions.

We asked ourselves the following questions: Do those students who correctly answer the multiple-choice questions carry out reasoned actions in open problems? Are their actions a real, convincing indication of having had a solid basis when answering the multiple-choice questions correctly?

Our belief is that most students who choose the correct answers in multiple-choice questions do not appear to do so on a solid basis.

Our analysis could possibly highlight elements for reference to clear up various positions that are being constantly assumed in the debate on which forms of assessment are the most appropriate when examining the understanding of statistical concepts. In this respect we will cross reference results of pairs of interrelated problems.

Methodology

Sample

Our study was undertaken with 94 students in the final year of secondary education, their average age being 17 years old. Throughout their schooling they had received specific instruction in arithmetical averages and other topics concerning statistics and probability.

Questionnaire

In this work we put forward data regarding four questions that make up a questionnaire of seven questions and that form part of a wider study that we are carrying out on the assessment of the concept of average. The four questions, as shown below, are made up of both multiple-choice and open-question items, which we designate according to the context defining them.

Question "In One Class": *A teacher decides to study how many questions her students do. The questions done by her 8 students during one class are shown below*:

	Names of students							
	Juan	Lucía	Alberto	Ana	Pedro	María	Luis	Clara
Nº of questions	0	5	2	22	3	2	1	2

The teacher would like to summarise these data, calculating the typical number of questions done that day. Which of the following methods would you recommend?

(Mark one of the following answers)

_______a. *Use the most common number, which is 2.*

_______b. *Add up the 8 numbers and divide by 8.*

_______c. *Discard the 22, add up the other 7 numbers and divide by 7.*

_______d. *Discard the 0, add up the other 7 numbers and divide by 7.*

This question is taken from Garfield (2003) with some modifications in the text. It is used in order to try and examine students' knowledge of averages, the use of the average calculation algorithm, the effect of atypical values, as well the importance of the context.

Question "Children per Family": *The school committee of a small town wishes to find out the average number of children per family in the town. They divided the total number of children by 50, this being the total number of families. If the average if 2.2 children per family, which of the following statement do you agree with?*

_______*a. Half of the families in the town have more than 2 children.*

_______*b. In the town there are more families with 3 children than with 2 children.*

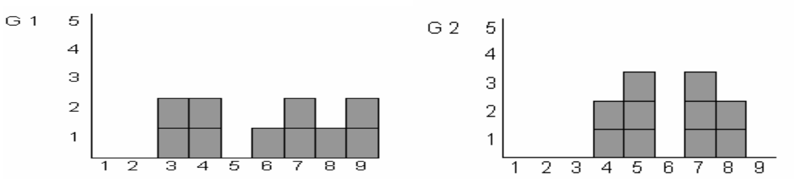
_______*c. There is a total of 110 children in the town.*

_______*d. There are 2.2 children per adult in the town.*

_______*e. The most common number of children in one family is 2.*

This question is also taken from Garfield (2003). Here the aim is essentially to evaluate understanding and proper use of the arithmetical average calculation algorithm.

Question "Marks Graph": *Twenty high-school students take part in a mathematics competition. Ten of the students form Group 1 and the other ten Group 2. The marks they achieve in the competition are shown in the graphs below:*



Each rectangle in the graph represents the marks achieved by each individual student. For example, in Group 1 the two rectangles appearing above Number 9 show that two students in this group achieved a score of 9.

5.A *Group 1 has an average mark of 6.*

a) Check that the average mark for Group 2 is also 6.

b) Which group seems better to you? Justify your choice.

5.B *Which of the following statements is true?*

_______*a Group 1 is better than Group 2 because the students who got higher marks are in this group.*

_______*b Group 2 is better because there are no students with marks below 4.*

_______*c There is no difference between the two groups because the average is the same.*

_______*d Although the averages are the same for both groups, Group 2 is more homogeneous.*

This question is of our own devising, although there is some similarity to one described by Garfield (2003). The aim of this question is to see how students interpret distributions shown in the form of a graph, find out if they know how to manipulate data graphically in order to calculate and examine what criteria they use when checking two samples based on their visual appearance.

Question "Family": *The average family size in a given locality is 3.2 persons. Show 10 families that fulfil this average.*

This question was adapted from Mokros and Russell (1995) and is designed to evaluate whether students are able to construct a distribution where the average is known. Also, we wish to check the strategies used by the students to find the distribution asked for, assess their understanding of the type of data to be used given the context, seeing that the average cannot form part of the distribution.

Categories

We set out a system of categories in order to codify students' answers, which would let us treat the information using statistical software.

The categories were established taking into account the type of item. For multiple-choice answer types we related the category to the mathematical content of the distracter, while for open answers we put forward as a category the strategy used by the student when solving the problem.

Below we give the codes that are directly related to the results given in this work:

Code	Code description
ADALG -	Adds up all the numbers and divides by the total of data.
DATIP -	Discards the value considered atypical, adds up the other 7 numbers and divides by 7.
DCERO -	Discards the 0, adds up the other 7 numbers and divides by 7.
MODA -	Uses the most common number.
ALG-SI -	Uses the simple arithmetical average algorithm.
ALG-SI-e -	Uses the simple arithmetical average algorithm, but incorrectly
ALGPOND -	Uses the weighted average algorithm correctly.
ALGPOND-e -	Uses the weighted average algorithm incorrectly
NC -	No answer.
SC -	Undertakes incoherent transformations or puts forward confused justifications or chooses more than one option.
SOPER -	Gives a numerical value without showing operations.
SX -	Adds up the four values on the horizontal axis on which the rectangles are built and divides by four.
STOTAL -	Indicate the option that says that there is a total of 110 children
DINCOR -	Gives a distribution that fails to fulfil the conditions of the question text.
DSOPER -	Gives a correct distribution, but does not show operations.
DSTOTAL -	Gives a correct distribution, attaining the sum total.
IGUMEDIA -	Justifies that the groups have the same averages or chooses the statement that indicates that there is no difference between the groups because the averages are the same.
MHOMO -	Puts forwards a justification based on homogeneity or chooses the statement referring to this criteria.
MNOTA -	Uses as criteria for justification the greater mark factor or marks the option referring to this criteria.

N<4 -	Uses as an argument the fact that there are no marks less than 4.
NAPROB -	Uses as justification the fact that there are more students who pass or marks the option referring to this factor.
NJUST -	Indicates that one group is better, but without justifying this idea.

Results and discussion

We shall basically analyse the actions in open questions of those students who chose the right answer in the multiple-choice questions.

Table 1 shows the data referring to the cross referencing of results for the question "Marks Graph, 5.A.a)" (rows) and "In One Class" (columns).

	ADALG	DATIP	DCERO	MODA	Total
ALG-SI	4	0	2	0	6
ALG-SI-e	1	0	0	0	1
ALGPOND	23	5	11	2	41
ALGPOND-e	1	1	0	0	2
NC	8	3	6	0	17
SC	1	0	3	2	6
SOPER	6	3	4	1	14
SX	2	1	3	1	7
Total	46	13	29	6	94

Table 1 Cross referencing of results for the **open question** "Marks Graph", 5.A.a (rows) and **multiple-choice question** "In One Class" (columns)

The data show that 46 (49%) students answered correctly (ADALG) the multiple-choice question while 47 (50%) also answered correctly the open question (ALG-SI and ALGPOND). The correct choice in the multiple-choice question is reached by adding up all the values and dividing the sum by the number of data, this being the average calculation algorithm.

Of the 46 students who chose the correct option in the multiple-choice question, when given the open problem that required them to interpret the data from the graph before using the average calculation algorithm, 19 (41%) students were unable to attain the solution that was required of them. Of these, 8 (17%) students did not answer (NC), 6 (13%) supplied a numerical result without showing the pertinent operations (SOPER), 2 students added up the values of the variables without taking into account the frequencies, 2 used the average calculation algorithm incorrectly, and 1 used incoherent procedures. The right answer was reached by 27 (59%) students who used two different procedures: 4 (9%) students used the simple average algorithm. (ALG-SI) and 23 (50%) used the weighted average algorithm (ALGPOND).

On the other hand, the data show that of the 47 students who answered the open question correctly 20 (43%) of them had indicated the wrong option in the multiple choice question (DATIP, DCERO and MODA).

These data seem to suggest that those students who marked the correct answer in the multiple-choice question merely considered that in those circumstances they could use the average calculation algorithm but when they were given a concrete situation they did not know how to use that instrument, perhaps because they did not know how to determine the various elements that make up the formula for calculating averages, a difficulty already underlined by Pollatsek, Lima and Well (1981).

Table 2 shows information taken after cross referencing the results for the questions “Family” (rows) and “Children per Family” (column).

	FAMADUL	MED	MODA	SC	STOTAL	Total
DINCOR	0	0	2	0	2	4
DSOPER	1	2	10	0	8	21
DSTOTAL	0	0	20	0	6	26
NC	1	2	14	0	14	31
SC	1	1	6	1	3	12
Total	3	5	52	1	33	94

Table 2: Cross referencing of the results for the **open question** “Family” (rows) and **multiple-choice question** “Children per Family” (columns).

A total of 33 (35%) students correctly answered the multiple-choice question (STOTAL), while for the open question 47 (50%) students correctly attained the solution required (DSOPER and DSTOTAL). Although attaining the correct solution for both these items depended on the same strategy (inversion of the average calculation average), the data show that of the 33 students who chose the correct answer in the multiple-choice question, only 14 (42%) were capable of solving the open question, while the other 19 (58%) had various difficulties: 14 (42%) did not answer (NC), 2 students showed a distribution that failed to fulfil the requirements stipulated in the question text (DINCOR) and 3 gave incoherent algebraic transformations (SC).

Of the 47 students that answered the open question correctly, it is noteworthy that 30 (64%) appear to have confused the average with the mode, as they preferred the distracter related to this concept in the multiple-choice question.

We were greatly surprised by the fact that most students who correctly identified the solution to the multiple-choice question could not even demonstrate the initial steps in solving the open question, in spite of the fact that the methodology for solving the two questions had the same basis. This leads us to believe that these students failed to use any criteria when selecting the correct answer.

Use of the strategy of inverting the average calculation algorithm was a basic step in solving the two items. As Watson and Moritz (2000) point out, this procedure was the main way of successfully solving a similar problem when studying the intuitive meaning given by children to the term “average”. However, for our open question in particular, it was crucial that students understood the ideas of distribution and average as an even sharing, as well as knowing the calculation algorithm, as explained by Cobo and Batanero (2004). It should also be pointed out that there was a further difficulty in that students were unable to use decimal numbers as data when constructing the distribution, due to the context. Possibly, then, most of these students found it difficult to solve the open question.

Finally, we show the cross referencing of results for the items “Marks Graph, 5.A.b)” (rows) and “Marks Graph 5.B” (columns).

	IGUMEDIA	MHOMO	MNOTA	N<4	NC	SC	Total
IGUMEDIA	13	1	0	0	0	0	14
MHOMO	0	6	0	0	0	1	7
MNOTA	2	1	4	0	0	0	7
N<4	0	1	0	1	0	0	2
NAPROB	5	14	1	14	3	6	43
NC	1	6	0	0	0	1	8
NJUST	0	1	1	0	0	0	2
SC	2	6	2	1	0	0	11
Total	23	36	8	16	3	8	94

Table 3: Cross referencing of results for the **open question** "Marks Graph, 5.A.b)" (rows) and **multiple-choice question** "Marks Graph 5.B" (columns).

Table 3 shows that 36 (38%) students marked the correct justification for the multiple-choice question (MHOMO), where this option referred to homogeneity and indicated the need to take into account the dispersion of data when comparing groups with the same averages. With respect to the open question, we could only find 7 (7%) students who gave the correct justification (MHOMO) using the same criteria. Also, of the 36 students who chose the correct option in the multiple-choice question, 30 (83%) had used incorrect arguments for the open item, that is, only 6 (17%) students gave acceptable arguments. The baseless justification most in evidence was the one that took for its criteria of comparison the fact that there were more students passing (NAPROB), given by 39% (14) of these students. We also found that 6 (17%) students who failed to answer (NC), another 6 (17%) who put forward confused justifications (SC), 1 who said that there were no differences between the groups because the averages were the same (IGUMEDIA), and 2 who only looked at the maximums or minimums of the distribution (MNOTA and N<4).

With regard to those students who correctly justified the open question, the data show that nearly all of the students also correctly got the answer to the multiple-choice question, except 1 (SC).

These results lead us to suspect that those students who chose the correct answer for the multiple-choice question did so without taking into account a formal basis, since, after choosing the answer, they did not bother to rectify the wrong justifications they had put forward in the open question. This also shows that the students are not consistent in their affirmations. In the same situation they use completely different criteria!

The difficulties arising when comparing samples in which students merely analyse only one part of the distribution of the maximum or minimum values were also found by Godino and Batanero (1997), and Estepa and Sánchez (1996). As interpreted by Estepa and Sánchez (1996), these difficulties are due to the fact that students have a local concept of association of variables and believe that this is the analysis which explains the differences between the two samples.

Conclusions

Our study has allowed us to see that many students who choose the correct answer for the multiple-choice questions are not able to demonstrate reasonable methods for solving open questions. The actions in open questions by those students who choose the correct answer in the multiple-choice questions, suggest that they choose these answers without any criteria. As can be seen in the results, especially in Table 2, most of these students are unable to follow the first steps in solving the open question, in spite of the fact that the solution to this question requires the same strategy that they should supposedly use before choosing the correct answer for the multiple-choice question. Another significant fact can be seen in Table 3, where students do not even remember to change the wrong justification they put forward for the open question after choosing the correct answer for the multiple-choice question.

The results also show incoherence in students' actions when they correctly mark the answer to the multiple-choice question and are unable to solve a related open question or vice versa. Furthermore, the results show that students are not consistent in their affirmations, given that in the same situation they use completely different criteria.

As can be seen in this study, the difficulties and incoherence evident in students' actions have been detected through the use of open questions, underlining the importance of this type of question when assessing the concepts held by students.

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