A TEACHING EXPERIENCE WITH LINEAR PATTERNS AND SOCIOMATHEMATICAL NORMS

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Abstract: In this paper we report briefly on our research on students' modes of argumentations during a teaching experiment on linear pattern generalizacion. During the teaching experiment the implementation of social and sociomathematical norms probed useful in helping students to develop a deeper understanding on pattern generalization and modes of argumentation to validate their and others findings.

Introduction

The importance of working with numerical patterns in compulsory secondary education has became a proposal of the current reform in many countries. For instance, a well known publication of the National Council of Teacher of Mathematics (N.C.T.M., 1989) states that " *represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations*" should be included in the mathematics curriculum for grades 9-12.

The new Spanish mathematics curriculum in compulsory secondary education states that "*Identificar y describir regularidades, pautas y relaciones conocidas en conjuntos de números y formas geométricas similares*" should be an assessment criteria (Real Decreto, 1991).

However, if you accept this statement there are many questions which need answering :

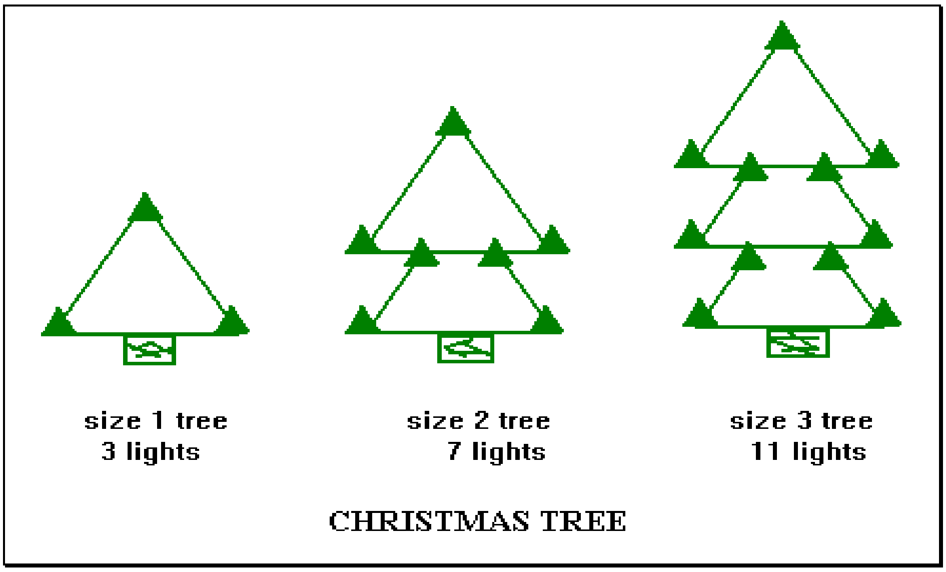
1) To which extent is it reasonable to expect every secondary student be able to recognise and work with patterns ? Is there a general pattern ability or competence?
2) Which should be the focus when teaching numerical patterns? To develop a numerical or algebraic competence? To be able to express a pattern using words?
3) What should be the students' and teacher's role and classroom atmosphere?

The Spanish secondary compulsory mathematical curriculum states also that for students following the Math-A curriculum (which includes the low-attainers) formalizations, unless neccesarly, should not be a focus of learning.

Our classroom experiment on teaching numerical patterns consisted of a group of low-attainers students, aged 15 to 16, in the last year of compulsory secondary education.

At the begining of the teaching experience, a problem task was administered to the whole group to gather information about students' previous knowlege on the topic. This task consisted in a word problem describing a sequence of objects (drawing included) which underlies a linear pattern, i.e., an affin function $f(n)= an+b$, a and b whole numbers and $b \neq 0$.

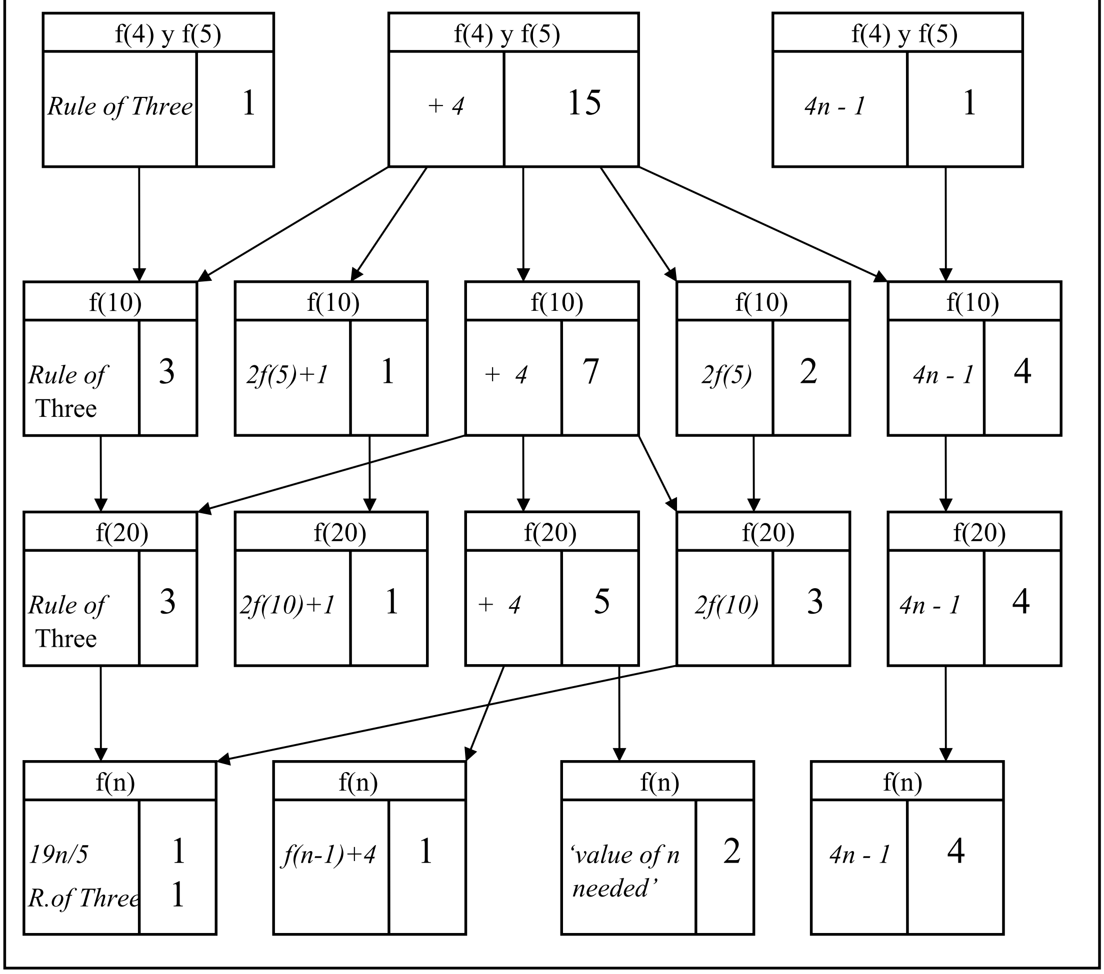
The previous task



a)How many lights are there on a size 4 tree?
b)How many lights are there on a size 5 tree?
c)How many lights are there on a size 10 tree?
d)How many lights are there on a size 20 tree?
e)How many lights are there on a size n tree?
Explain how you found your answer.

The students' responses to this previous task were classified and the results are displayed in the GVT (Graph for Visualizing Transitions) which is a visual tool we have developed during our resarch in linear generalizing problems (García Cruz & Martinón, 1997b)

GVT for Christmas Tree problem
Strategies and frecuency of students' responses.



Aims and methodology

Due to the lack of pattern's competence showed by students in the preliminary task, only 5 students out of 17 were able to give a correct and generalizable strategy, the objetives of the teaching experiment were set up as follows:

1) to develop a deeper understanding relating the individual solutions to the drawing accompaning the problems.

In our research we have show that the drawing accompaining the questions plays a twofold role in the process of abstractions and generalization (García Cruz & Martinón,1997a), being the setting were the generalization is established for the students developing a visual strategy and also the setting were the students check the validity of the numericla strategy developed during the process of abstractions and generalization.

2) to focus the students' performance, not only on arithmetical or algebraic competence but in developing also communicating abilities to express and to judge their own and others' findings.

These objetives were set up with the aim of negociating with students to norms:

First the social norm in which students are expected to explain their solutions and their way of thinking and second, the sociomathematical norm in which they are expected to contribute to the whole-class discussion with different solutions and also to judge the solutions provided by others students (Yackel and Cobb, 1996).

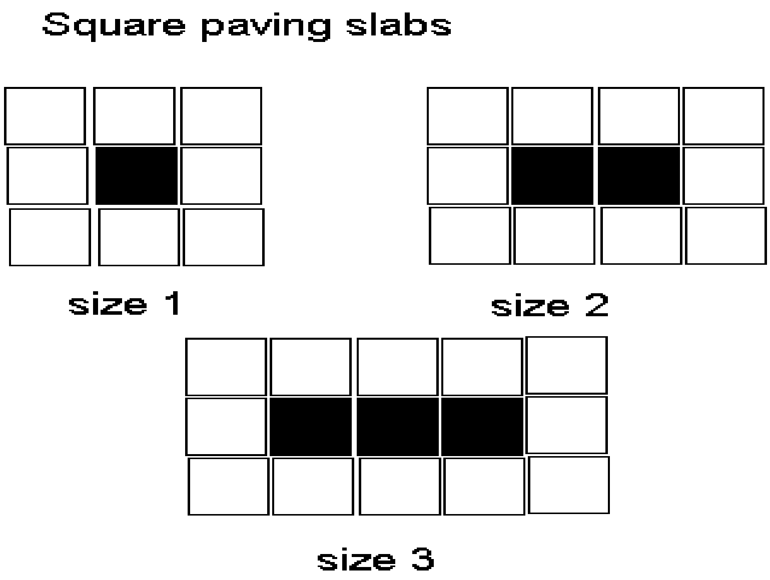
A set of three new related activities were developed to use during the classroom sessions and instruction basically consisted of teacher-led discussion of problem posed in a whole-class setting, collaborative small-group work between pairs of students, and follow-up whole-class discussion where students explain, justify and judge the interpretations and solutions they develop during small-group work.

So the teacher's role was to facilitate and encourage students' participation in small-group and whole-class discusion. When a strategy was explained he had to ask for any other students who wanted to judge the strategy, not showing disagreement or any kind of behaviour which could give any hint to the whole group about the correct or incorrect quality of that strategy.

Students were obliged to try to develop personally meaningful solutions that they could explain and justify, and reflection upon their own and others' strategies of solutions was encouraged.

Some excerpt from the teaching sessions

The first task posed to the whole-class was '*square paving slabs'* .



A paper copy of the task was given to any student and then the following question was asked : *How many white slabs will you need to surround a square paving of size ten ?* . After some students' work the teacher choose some incorrect approach to allows whole-class discussion. The choose of an incorrect strategy allows a better whole-class discussion, because the holder has to explain its validity and at the same time he/she is exposed to the whole-class questioning. During our experiment this approach was very successful. And after another two similar task we have three different strategies developed and fully discussed and three different general schemata reached.

To illustrate these schemas we will refer each to the *'square paving slabs'* situation.

First schema (Julio) :	$f(10) = 2 \cdot (10-1) + 8$	$f(n)=2(n-1)+8$
Second schema (Ana) :	$f(10) = 2 \cdot 10 + 6$	$f(n)=2n+6$
Third schema (Samuel) :	$f(10) = 3 + 10 + 10 + 3$	$f(n)=3+n+n+3$

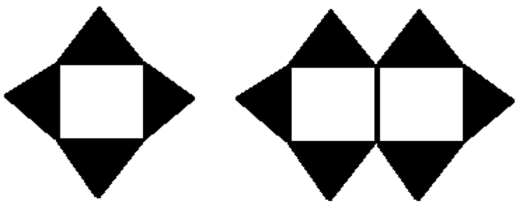
Then the three schemata were simbolized and also simplified and as a result the symbolized expresion for Ana's schema was obtained. This algebraic task is very important because through it the equivalence of the three schemata are highlighted, and also the specific relationship connecting any particular action (numerical or pictorical) within a strategy with the abstract symbols used.

We had stressed during the class session the students' understanding that when discussing a problem they should offer solutions different from those already contributed, which is a social norm, but what constitutes mathematical difference is a sociomathematical norm. When discussing the validity or difference of a strategy the students had to contribute with specific arguments highlighting *how* a strategy or schema is different and *why* it is correct, and in doing so students have to reflect on their own thinking and to find out appropriate arguments. So these norms regulate mathematical argumentation and influence the learning opportunities for the whole class.

A final task was posed at the end of the teaching experiment, in order to have written responses of the students and to allows a better and deeper analysis of the students' arguments. This task consisted of four different given solutions to a problem and the students had to develop and explain their own solution and to compare their own solutions with the four given, to judge differences and similarities and to question the validity of these solutions using appropriate arguments.

The final written task

Look at the following drawing:



To enclose a square we need of four triangles
to enclose two adyacent squares we need of six triangles.
The teacher has posed the following question :
How many triangles are needed to enclose 30 adyacent squares ?
below are the answers given for four students :
José: $2 \times 30 = 60$, $60 + 4 = 64$. *Answer : 64 triangles are needed.*
Luisa: $30 + 30 = 60$, $60 + 1 + 1 = 62$. . *Answer : 62 triangles are needed.*
Jonathan: two squares need of 6 triangles so 30 squares need of 90 triangles, because $30 = 2 \times 15$ *y* $15 \times 6 = 90$.
Ana: one square need of 4 triangles so 30 squares need of $4 \times 30 = 120$ *triangles.*
Questions :
a) Which solution from the four given is correct ?
b) Explain why the others are incorrect.

Let us state here some remarks about the given solutions to final task. The whole set of four strategies were similar to some students' contribution during the class experiment, only one is right and the other three underlies a mistake as is explained below.

The José's given solution is a mistake based on the concurrece of both Ana and Julio strategies stated above. The variable component is taken from the Ana's strategy while the constant term is taken from the Julio's strategy.

The Luisa's solution is right and correspond with the schema of Samuel stated above, which is strongly based on the drawing.

The Jonatha's solution is not a very common found mistake. The second picture tells us that two squares needs of six triangles, and it is assumed that an object of size 30 is equivalent to 15 objects of size 2, so f(30)=15 f(2)= 15×6 = 90.

Finally the Ana's given solution underlies a common mistake in that kind of problems. The students at this level discover very easy that the pattenr consist in adding the common difference to any term in order to obtain the following one, so implicity is assumed that repeated addition of 2 implies multiply by 2.

In explaining the above questions four types of arguments were used by students :

A0 : The validity of the given solution is stated by comparation with the calculation performed by the student. So if the two numbers are equal then the solution is correct, otherwise is incorrect. The argumentation is vague and not accurate.

A1 : The argument basicaly repeat itself the calculation process of the given solutions and no other reasons are given.

A2 : The argument is based in a counterexample to invalidate the incorrect solutions given.

A3 : The explanations shows that the student has understood the mathematical operations given. The drawing o the numerical sequence is used to validate the calculations performed.

The frecuency of students' arguments is displayed in the following table.

Type of Argument used for each given solution				
	José	**Jonathan**	**Ana**	**Luisa**
A0	5	2	1	1
A1	2	5	4	3
A2	1	1	6	--
A3	9	5	5	13
no answer	1	5	2	1

We have classified arguments A2 and A3 of a higher-level-mode. Students using these types of arguments shown an aceptable level of comprehension of the solutions given, thus these arguments are quite related to the relational understanding (Skemp, 1976). Otherwise the arguments A0 and A1 are quite related to the instrumental understanding and thus they should be classified as belonging to a low-level-mode.

We can see in the following table the frecuency of use of each mode to any given solution:

Arguments	***José***	***Jonathan***	***Ana***	***Luisa***
low-level- mode	7	7	5	4
high-level-mode	10	6	11	13

The use of high-level-mode arguments overcame the use of low-level-mode arguments in any given solution except *Jonathan's* solution which is nearly the same.

Another question of interest is the consistency of choice of argument. The following table display that choice for any of the task questions given broken down by students:

Student	**José**	**Luisa**	**Jonathan**	**Ana**	**nº arg.**	**A3&A2**	**A1&A0**
Jonathan	A3	A3	A3	A3	1	4	0
Julio	A3	A3	A3	A3	1	4	0
Jonay	A3	A3	A3	A2	2	4	0
Airám	A3	A3	A1	A1	2	2	2
Samuel	A0	A3	A0	A3	2	2	2
J.Antonio	A3	A1	A1	A1	2	1	3
Beatríz	A2	A1	A3	A2	3	3	1
Pedro	A0	A3	A2	A2	3	3	2
Yisel	A1	A3	A1	A2	3	2	2
Iván	A3	A3	A1	A2	3	3	1
Ana Rosa	A3	A3	-	A3	1	3	0
Marcos	A0	A3	-	A3	2	2	1
Carlos	-	A3	A1	A1	2	1	2
Yaiza	A1	A3	-	A1	2	1	2
Tamara	A0	-	A0	A2	2	1	2
Aranza	A3	A0	-	A0	2	1	2
Santiago	A0	A3	-	-	2	1	1
Bibiana	A3	A1	-	-	2	1	1

From the table we can see that only 4 students do not use a low-level-mode argument and all students use a high-level-mode argument at least, although only four students are consistent with the use of arguments belonging to the same mode.

The results show that many students have developed a good understanding about what counts mathematically with regards to mathematical arguments, but this is not a level achieved generally, and many students still rely in a naive way of judgement and many explanations are on the way of explaining '*what it is being done'* instead of '*why it is being done that way'*.

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